ON THE PROBLEM OF SIMILE CONFIGURATIONS

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On the validity of the vectorial formula $\overline{P} = \lambda\overline{Q} + \overline{R}$ in regard to a special problem of homographich solutions of the three-body problem with law of attraction proportional to the inverse of the cube of the mutual distances between the masses.

1. In a meeting held at Córdoba Observatory in 1961, Dr. R.P. Cesco objected some results I had obtained in dealing with a problem of homographic solutions of the three-body problem with law of attraction inversely proportional to the cube of the mutual distances between the three bodies (Boletín de la Asociación Argentina de Astronomía, N° 4, 1961).

In that opportunity Dr. Cesco argued that he had proved in his paper entitles "SOBRE LAS SOLUCIONES HOMOGRAFICAS DEL PROBLEMA DE LOS TRES CUERPOS" Pub. del Obs. Astr. de La Plata, Tomo XXV, N 2, 1959, the existence of a scalene triangular configuration in this problem. Moreover, due to this result Dr. Cesco announced in the PROCEEDINGS OF THE INTERNATIONAL MEETING ON PROBLEMS OF ASTROMETRY AND CELESTIAL MECHANICS, Pub. Obs. Astr. de La Plata, 1961, that a theorem proved by Wintner is wrong. In this theorem Wintner stated that a particular triangular homographic solution obtained by Banachiewitz (1906), must be isosceles.

In order to demonstrate the inconsistency of the results obtained by Dr. Cesco in the aforementioned papers, I aim to show that Dr. Cesco's vectorial formula $\overline{P} = \lambda\overline{Q} + \overline{R}$, where $\overline{P}$ and $\overline{Q}$ are vectors of constant modulus and functions of the time, $\overline{R}$ is a constant vector, λ is a positive scalar, is not compatible with the assumptions made by this author in order to solve the problem. It must be emphasized that Dr. Cesco's vectorial formula is the "kernel" upon which he bases his results.

2. The main question arises from the fact that Dr. Cesco has evidently changed the definition of an homographic solution of the problem of three bodies. In fact, he adopted ordinary vectors for the statement of the problem. This way of defining an homographic solution is different from the classical definition used in dealing with this problem. Classical theory shows that in order the vector products (i.e. scalar product and vectorial product) be equivalent to the effects produced by matrices of rotation in the Euclidean three dimensional space, vectors must transform into tensors.

From this point of view it will only be necesary to demonstrate that the results obtained from Dr. Cesco's definition, are not compatible with the statement of the problem.

PROOF Let us suppose that the vectorial equations of motion of three bodies are given. If an "heliocentric" system (rectangular) of reference is set up, being the origin at the mass m_o, say, we have for the special law of the inverse cube of the mutual distances:

(1)
$$\rho^4\ddot{\overline{r}}_1 + a_{11}\overline{r}_1 + a_{12}\overline{r}_2 = 0$$
$$\rho^4\ddot{\overline{r}}_2 + a_{21}\overline{r}_1 + a_{22}\overline{r}_2 = 0$$

$$a_{11} = (1 - m_2)\, a_1^{-4} + m_2 a^{-4} \quad ; \qquad a_{12} = m_2 \; (a_2^{-4} - a^{-4})$$

$$a_{22} = (1 - m_1)\, a_2^{-4} + m_1 a^{-4} \quad ; \qquad a_{21} = m_1 \; (a_1^{-4} - a^{-4})$$

Remark This is the form that the equations appear written in Dr. Cesco's aforementioned papers. We shall see now the meaning of the scalar ρ .

3. Following Dr. Cesco's statement we write

(2) $\bar{r}_1 = \rho \bar{P}$ $\quad \bar{r}_2 = \rho \bar{Q}$, $\bar{r}_1$ and $\bar{r}_2$ are vectors of position of the masses m_1 and m_2 respectively, respect to the mass m_o. $\bar{P}$ and $\bar{Q}$ have already stated, and $\rho(t)$ is the dilatation, such that $\rho(t) > 0$

In order to transform equations (1) by means of equations (2) Dr. Cesco introduces a new independent variable τ such that $d\tau = dt/\rho^2$. In this way there will be a system of differential equations of second order in the $\bar{P}$'s and $\bar{Q}$'s. After some elementary operations Dr. Cesco arrives at the following relationship:

$$\bar{P}' = \lambda \bar{Q}' \;, \text{ where } \bar{P}' = \frac{d\bar{P}}{d\tau} \;, \; \bar{Q}' = \frac{d\bar{Q}}{d\tau}$$

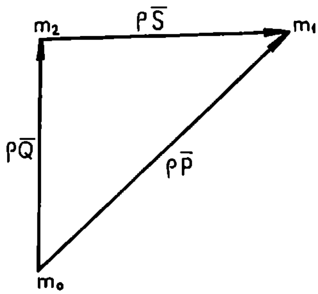
Vectors $\bar{P}'$ and $\bar{Q}'$ are according to Dr. Cesco's results perpendicular to the plane formed by the three bodies. On the other hand we easily obtain also from $d\tau$'s definition:

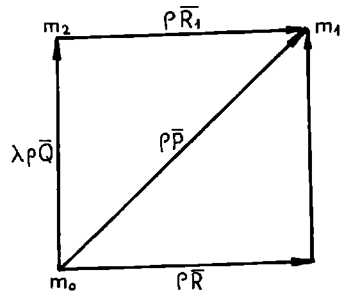
$$\dot{\bar{P}} = \lambda \dot{\bar{Q}} \quad , \text{ where } \dot{\bar{P}} = \frac{d\bar{P}}{dt} \;, \; \dot{\bar{Q}} = \frac{d\bar{Q}}{dt}$$

By a simple quadrature it is obtained

(3) $\bar{P} = \lambda \bar{Q} + \bar{P}$, where $\bar{R}$ is constant vector of integration.

4. Let us investigate the compatibility of this vectorial formula with the one which gives the configuration of the three bodies such as it has been stated in the hypothesis of the proble. To do this we multiply formula (3) by the scalar ρ . Drawing the vectors of position in both cases we obtain the following figures:





(4) $\rho \bar{P} = \rho \bar{Q} + \rho \bar{S}$ $\qquad \rho \bar{P} = \lambda \rho \bar{Q} + \rho \bar{R}$

It is clearly seen that in order that the mass m_2 have the same vector of position in both cases we must have $\lambda = 1$. In this way it is quite clear that Dr. Cesco's homographic solutions for $\lambda > 1$ and for $\lambda < 1$ must be disregarded. In order to complete our discussion, we see that for $\lambda = 0$ there is no homographic solution, because it would imply a division by zero.

The last case for $\lambda = 1$, can be trated as follows. Putting $\lambda = 1$ in the second of equations (4), the comparison of both figures gives that vectors $\rho \bar{S}$ and $\rho \bar{R}_1$ are equal. But vector $\rho \bar{R}_1$ is equipolent to vector $\rho \bar{R}$, and $\bar{R}$ is a constant one. That would imply that $\bar{S}$ is a constant vector too. This contradicts the hypothesis.

There are two another facts to be remarked:

1) On account of the results attained by Dr. Cesco the plane of the three bodies cannot rotate in the three-dimensional space, because vectors $\bar{R}$ and $\bar{R}_1$ are constant, and because also the barycenter is fixed (we must remember that the system of reference is an "heliocentric" one, and so, the degree of freedom of the system of differential equations has been reduced). Being fixed the plane formed by the three bodies, vectors $\dot{\bar{P}}$ and $\dot{\bar{Q}}$ must be null. From this we conclude that

$\bar{P}$ = const. $\bar{Q}$ = const., which contradicts hypothesis.

2) The second remark shows a very interesting fact. All the problem deals with central configurations. That is to say that the acceleration must be in the direction of the barycenter. But this is not the case for the mass m_o in Dr. Cesco's vectorial solutión. This mass belongs to a constant vector $\bar{R}$, so it can only move in this direction.

That means that there is no component at all of the acceleration in the direction of the barycenter; this mass cannot have a central movement. With this proof we have completed the discussion which has shown that Dr. Cesco's solutions are not valid. Moreover as Dr. Cesco has used this results in trying to demostrate that Wintner's afore-mentioned theorem is wrong, it is clear that in regard to this problem, Dr. Cesco's results are again untenable and therefore, Dr. Wintner's theorem holds true.

EL CUMULO ABIERTO ALREDEDOR DE LA ESTRELLA η CARINAE

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Las observaciones fotoeléctricas efectuadas durante el presente año (1964) de las estrellas situadas en un radio de 5' alrededor de η Carinae muestran que todas ellas son miembros del cúmulo abierto Trumpler 16. La estrella más brillante del grupo, HD 93250, de tipo espectral 05, se encuentra sobre la secuencia principal de edad cero. Su magnitud absoluta es entonces $M_V = -5.5$.

La estrella Wolf-Rayet, HD 93162 sería un miembro del cúmulo, siempre que se admita, además de la absorción interestelar de todo el grupo, una absorción adicional de $A_V = 0^m.6$. Con un exceso de color de $E_{B-V} = 0^m.2$ esta estrella estaría muy cerca de HD 93250 en el diagrama color-magnitud.